8-3-2020

"Teaching Them How to Fish": Learning to Learn and Teach Responsively

Caroline B. Ebby PhD
University of Pennsylvania, cbe@gse.upenn.edu

Brittany Hess
University of Pennsylvania, brhess@upenn.edu

Lizzy Pecora
University of Pennsylvania, lpecora@upenn.edu

Jennifer Valerio
University of Pennsylvania, valerioj@upenn.edu

“Teaching Them How to Fish”: Learning to Learn and Teach Responsively

Abstract

The Responsive Math Teaching (RMT) project’s 3-year model for professional development introduces teachers to a new instructional model through a full year of monthly Math Circles, where they experience problem solving and productive struggle from the student perspective while working through challenging open-ended tasks, engaging in mathematical discussions, and reflecting on the process. This paper examines teachers’ views of what they learned from this experience and how it affected both their instructional practices and their visions of mathematics teaching and learning.

This study focused on a group of 34 participants from a network of urban elementary schools who attended monthly after school sessions over the course of one academic year. We highlight key findings in relation to (1) what teachers learned from engaging in solving challenging math tasks as learners, (2) key elements of their developing visions of mathematics teaching and learning, (3) changes they reported to their math instruction. The evidence suggests that by the end of the introductory year, teachers were primed and ready to learn new skills and practices to help bring their developing visions into practice in the classroom.

Keywords

education, teacher learning, professional development, math, math education, mathematics, instructional vision, math classroom

Disciplines

Education

“Teaching Them How to Fish”: Learning to Learn and Teach Responsively

Responsive Math Teaching Project
Working Paper

Brittany Hess, Lizzy Pecora, Jennifer Valerio, and Caroline Ebby
Graduate School of Education
University of Pennsylvania

Contents

Abstract

The Responsive Math Teaching (RMT) project's 3-year model for professional development introduces teachers to a new instructional model through a full year of monthly Math Circles, where they experience problem solving and productive struggle from the student perspective while working through challenging open-ended tasks, engaging in mathematical discussions, and reflecting on the process. This paper examines teachers' views of what they learned from this experience and how it affected both their instructional practices and their visions of mathematics teaching and learning.

This study focused on a group of 34 participants from a network of urban elementary schools who attended monthly after school sessions over the course of one academic year. We highlight key findings in relation to (1) what teachers learned from engaging in solving challenging math tasks as learners, (2) key elements of their developing visions of mathematics teaching and learning, (3) changes they reported to their math instruction. The evidence suggests that by the end of the introductory year, teachers were primed and ready to learn new skills and practices to help bring their developing visions into practice in the classroom.

Key Findings

Experiencing Responsive Math Teaching as Learners

- Engaging in collaborative problem solving helped teachers understand the role of peer interaction in the learning process.
- Teachers came to value their own mathematical ideas and competency.
- Engaging in productive struggle helped teachers identify with their own students.

Developing Instructional Vision

- Teachers shifted their understanding of the teacher's role to include less direct instruction and more student-led learning.
- Teachers recognized the importance of building a common understanding, both of tasks and of other's mathematical ideas.
- Teachers believed in the importance of students taking on more of the cognitive load during problem solving.
- Teachers valued multiple strategies for solving problems and using them to illustrate mathematical connections.

Implementation of Responsive Math Teaching in the Classroom

- Teachers were more intentional in launching tasks.
- Teachers incorporated rich math tasks and opportunities for productive struggle into math instruction.
- Teachers had students share and make sense of each other's problem solving strategies.
- Teacher Leaders began to implement ideas from RMT into their coaching and classroom practice.

The Emerging Gap between Instructional Vision and Practice

- Teachers recognized that they needed more knowledge and new skills to teach mathematics responsively.
- Teachers had a new appreciation for the importance of planning out lessons carefully before teaching.
- Some teachers still questioned whether all students were capable of engaging in challenging problem solving and reasoning experiences.

> *Ambitious teaching requires that teachers teach in response to what students do as they engage in problem solving performances, all while holding students accountable to learning goals that include procedural fluency, strategic competence, adaptive reasoning, and productive dispositions. (Kazemi, Franke, & Lampert, 2009)*

Helping teachers learn new instructional practices that are responsive to student thinking is no small feat. Ambitious mathematics teaching can be more challenging than traditional prescriptive instruction because teachers must respond to both student thinking and mathematical learning goals in real time. Recognizing what students know and are able to do and then leveraging that foundation to move towards higher level reasoning and problem solving requires a different skill set, but such responsiveness is crucial for ensuring equity and access to mathematics for all students. Ambitious mathematics teaching recognizes every student as a capable learner who can develop deep, meaningful, and flexible mathematical understandings.

Professional development (PD) programs aimed at changing teaching practices often fall short of this goal for two main reasons. First, teachers' existing frames of reference, created over a lifetime of experience as both a student and a teacher, can make it difficult for them to envision how math instruction might be different. Second, even those who express interest in changing their teaching practice often lack the new skills required to do so (Kennedy, 1999; 2016). Short-lived, prescriptive professional development may provide momentary changes in teachers' attitudes or ideas but rarely moves the needle when it comes to what goes on in their classrooms.

To address this problem, the Responsive Math Teaching (RMT) project's innovative model for PD first offers teachers a full year of monthly Math Circles, where they experience responsive teaching from a learner's perspective while working through challenging high-quality tasks and engaging in rich discussions about mathematics. Equipping teachers with this solid foundational experience is meant to establish buy-in and provide a new frame of reference so that teachers are more invested in learning to enact responsive math teaching practices in their own classrooms. During the second year of RMT PD, teachers unpack, analyze, and rehearse RMT practices and begin planning and enacting responsive math lessons in their own classrooms. In the final year of RMT PD, participants use what they have learned from experience and enactment to take on a leadership role, helping others learn to teach responsively by providing model lessons, serving as coaches, and planning PD for their colleagues.

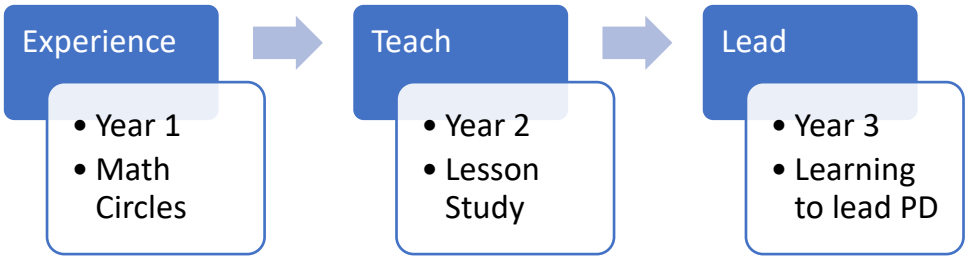

Figure 1. RMT Model of Professional Development

This paper examines participants' reflections at the end of their first year of involvement in RMT PD to explore what they learned from engaging in the Math Circle experience. More specifically, we explore how it influenced their views of learning and teaching mathematics as well as their classroom practice.

Context

The RMT Project is a research-practice partnership designed to build mathematics instructional leadership capacity in a network of 13 K-8 schools[1]. This learning network is situated within a large urban district where schools are largely segregated by race and socioeconomic status, and where there are considerable challenges in student achievement, human resources, and adequate funding. During the first year of this study, in 12 of 13 schools in this network, the percentage of students in grades 3-8 scoring proficient or advanced on the state assessment was 14%, compared with 21% in the district overall. In 11 of those schools, over 97% of students attending were classified as Economically Disadvantaged.

One of the more challenging aspects of developing capacity for instructional leadership is the high level of teacher turnover in both the network and the district. On average, 27% of teachers in the district leave their school at the end of each school year and half of those leave the district entirely (Steinberg, 2018). Due to this churn, some teachers and principals leave their roles and/or schools before completing the full three years of PD and support. To mitigate the effects of this turnover and build small learning communities, every school is encouraged to send a small group of teachers, rather than just one designated math leader, to RMT PD.

In 2016, the school district launched a new vision for mathematics instruction and set of guiding principles centered on the belief that, "all students think mathematically, and they will be empowered to own, share, and do mathematics." The guiding principles reflect commitments to "equitable discourse, rich and meaningful tasks, purpose-driven work, questioning and curiosity, and valuing diverse thinking" (School District of Philadelphia, n.d.). These commitments reflected a shift in the district's priorities away from a direct instruction or didactic model towards a more dialogic model centered around student thinking and ambitious instruction (Munter, Stein, & Smith, 2015). The district also adopted new mathematics curriculum materials and provided voluntary opportunities for professional development in the form of a week-long summer institute. Despite these investments, the focus of instructional improvement efforts and professional learning opportunities at the school level remained largely focused on early literacy instruction.

In collaboration with the district and network leadership, the RMT project developed and refined a model for math instructional leadership that builds on current research in mathematics education and provides sustained professional learning opportunities for teachers and leaders, through both out-of-school PD sessions and in-school classroom support. The overall goal of the research-practice partnership is to help build coherence throughout the different organizational levels of the instructional system by providing critical links to help translate district instructional vision into school and classroom practices.

One critical link is a model for mathematics instruction that describes the different components of a lesson, as well as specific teacher practices within those components, aimed at bringing students' ideas and the mathematical goal closer together. These practices include:

- Selecting and launching cognitively demanding tasks that are open to multiple entry points and solution strategies;
- Engaging students in productive struggle, discourse, and collaboration;
- Having students share strategies and defend their solutions;

[1] The district has a total of 342 public and charter schools that are divided into 17 learning networks.

- Intentionally building connections and deeper, more sophisticated understanding of core concepts, strategies, and procedures;
- Collecting formative assessment data to inform instruction.

The RMT Project provides monthly after school PD sessions in a cohort model over three years in order to furnish teachers with adequate time to experience, analyze, and practice learning and teaching through the RMT instructional model. This report focuses on teacher learning from the first year of professional development, a year-long series of Math Circle sessions where they experienced RMT as a mathematics learner but did not receive any formal in-school classroom support. During each PD session, the RMT facilitator launched an engaging and demanding mathematics task, facilitated participants' engagement in productive struggle, made their thinking visible, and connected their solutions to important mathematics. Each session ended with an opportunity to collaboratively reflect and debrief the experience in relation to the facilitator's moves, the learners' experiences and the RMT instructional model.

Methods

Of the 35 first-year participants in this study, 10 attended Math Circles in 2018-19 and 25 attended in 2019-20. Twenty-nine of these participants attended more than half of the after-school PD sessions offered. In 2018-19 there were a total of 8 sessions offered and in 2019-20 there were a total of 5 sessions before COVID-19 school shut-downs caused the PD to move to a virtual format. A few participants joined the sessions partway through their respective years. The participants came from thirteen different elementary schools. Their teaching experience ranged from 2 to 36 years with an average of 14.4 years (see Figure 2). Twenty-eight of the teachers taught in the classroom full time (grades 1 to 8) and seven of them were fully released math coaches (see Figure 3). While the participating teachers all worked in the same learning network, they came from schools with varying and diverse challenges (see Appendix A).

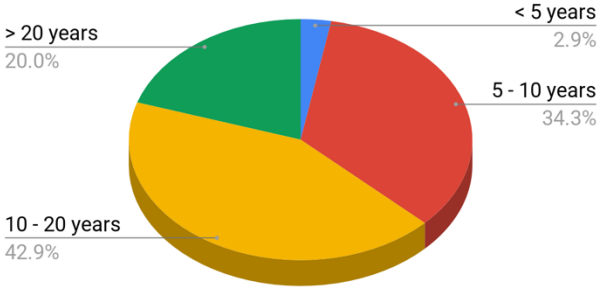

Figure 2. Participant years of experience.

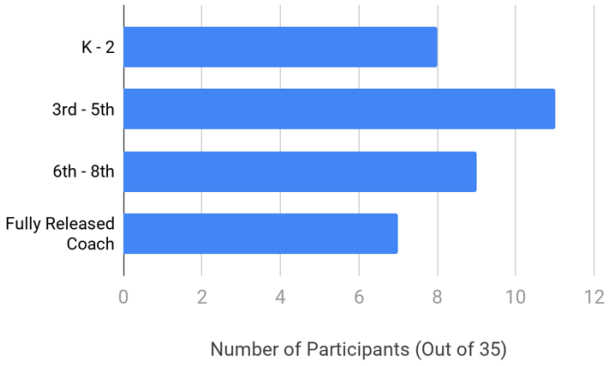

Figure 3. Grade levels taught by participants.

Data was collected through audio-recorded semi-structured interviews conducted with each participant in May of each year, after the majority of PD sessions had occurred. For this analysis, we focused on responses to questions that were directly related to participants' experience of the professional development; they were asked to describe the experience, what they learned, and what, if anything, they had applied to their own instructional practice.

All interviews were transcribed. The initial phase of analysis, which consisted of reading interview excerpts and identifying emergent themes, resulted in three major categories—experience as a learner, instructional vision, and classroom implementation. After organizing interview excerpts by those categories, we developed subcodes by looking for common themes that occurred across participants. Three members of the research team then coded excerpts from one category for evidence of those themes and reviewed each other's coding. Disagreements were reconciled through discussion to look for disconfirming evidence. To refine the data, we used a process of systematically organizing, coding, and narrowing down specific excerpts from the transcripts to construct a narrative of the findings in relation to each theme.

Findings

The first section explores what teachers learned from experiencing RMT as a learner. In the second section we examine how the PD experience influenced teachers' overall vision for what mathematics teaching and learning should look like. In the third section, we look at the changes teachers reported making in their classroom instruction even though this was not an expectation for the first year. Finally, we explore the challenges and goals teachers reported in relation to the gap they perceived between their developing vision and their own classroom or coaching practices.

Experiencing RMT as a Learner

Nearly all (33 out of 35) RMT participants reported having eye-opening experiences when they took off their teacher-hat and engaged in doing mathematics as a learner. Those notable takeaways fell into three categories: valuing the opportunity for peer interaction and collaboration, appreciating one's own mathematical ideas and capacity to do mathematics, and experiencing what it is like to be a student engaging in productive struggle.

- Engaging in collaborative problem solving helps teachers understand how peer interaction influences the learning process.

The majority of participants (80%), commented that working alongside a peer made problem solving more productive. If a teacher felt stuck in the process of solving a challenging problem, they found discussion and working with a group to be an effective means of getting unstuck. Participants also described having the opportunity to expand and deepen their mathematical understanding individually when they listened to their peers explain solutions or ways of seeing a problem that differed from their own. A first grade teacher expressed, "It kind of opened my eyes to the fact that there's no one way [to solve]." Ultimately, peer interaction provided support, encouragement, and deeper understanding for many participants. As one teacher noted, learning through peer interaction can also help to cement understanding so that it can be accessed more readily in the future:

> *I will remember not only what I did and what my answer was. I will remember the person that I worked with and what their input was because we had to work to get to it. It lasts longer. (Grade 6 teacher)*

- Teachers learned to value their own mathematical ideas and competency.

More than half of the participants described feeling like their voices and ideas were valued, because the facilitator never made them feel like the way they were solving a problem was wrong. This helped

participants feel empowered to draw on prior knowledge and experiences to get started on challenging problems. It also meant that participants were more compelled to contribute to small or whole group conversations. In general, receiving validation from the facilitator led to a deeper appreciation for mathematical thinking and reasoning processes.

> *Every time I left, there was something that I learned that I would never forget because I developed the knowledge on my own and no one gave it to me. So for me, it was like I own this because I struggled through it, that whole productive struggle. (Math Coach)*

A few participants who started off the year with low mathematical self-confidence expressed having some anxiety during the first few sessions. Over the course of the year, these participants became more comfortable with uncertainty and began to incorporate learned strategies to move past the initial feeling of intimidation to engage in the problem.

> *There are these moments in the very beginning where it's like, "Okay, now go and you figure it out," and I'm literally in panic because I see everybody writing...but I love when [the facilitator] asks something that triggers it and I'm like, I did kind of know. I think I was just fearful that I would be the one who didn't know. I think those [moments] stand out the most every time I leave...It's always like, ah, people love math for a reason. (Grade 1 teacher)*

As this quote illustrates, the experience of moving past the initial panic with the support of the facilitator helped teachers develop more appreciation for mathematical problem solving.

- Teachers experienced what it is like to be a student engaging in productive struggle.

Nearly half of the participants reported understanding their own students' feelings better after experiencing RMT as a learner. They described experiencing what it is like to be held accountable for their own work and to be frustrated when they could not immediately find a solution. Some mentioned the experience of being pressed by the facilitator to productively struggle, explain their thinking, or make sense of another person's idea. Others commented on the experience of having to actively participate in making sense of challenging mathematics content on a regular basis. At the same time, it helped them think about the role of the teacher. As one participant reflected:

> *I liked the PDs, because they gave me an opportunity to see what my students have to do… You know, it kind of gave me perspective on how they feel when they have to engage in a group. Seeing it from their point of view lets you know how clear and concise your directions must be. It makes you think things through more thoroughly when you're planning and when you're engaging with the students. (Grade 6 teacher)*

As this quote illustrates, teachers did not completely take off their teacher hat during the learning experience. Rather, they were always conscious of the facilitator's responsive actions and, consequently, reflected on their own teaching practices and instructional vision.

Instructional Vision

Instructional vision is defined by Cobb & Smith (2008) as teachers' statements about "what is important for students to know and be able to do mathematically...and how students' development of these forms of mathematical knowledgeability can be effectively supported" (p.7). These personal theories, or

aspirational images, are important because they have been shown to be predictive of teacher's take up of subsequent learning opportunities, supports, and resources (Munter & Correnti, 2017). Although the interview questions asked about what participants learned from their experience and what, if anything, they had tried in their classrooms, we noticed that teachers were articulating a fairly sophisticated vision for teaching and learning that incorporated some of these broader principles. The depth and breadth of their responses varied, but all but two of the participants made at least one statement that illustrated a shifting instructional vision based on their experience with RMT.

- Teachers shifted their understanding of the teacher's role to include less direct instruction and more student-led learning.

A third of the teachers mentioned an altered view of their own role in the classroom from their experience in the Math Circles. Teachers described decreasing their use of direct instruction, where students are told the necessary information, and instead allowing students to discover that knowledge themselves. As a first-grade teacher stated, "I learned that the teacher should walk around and help. You're not there talking. The learning is coming from the students." This quote reflects an important belief about learning; that it generates from the students and not from the teacher. Likewise, many teachers also indicated that learning should be student-led, allowing students the opportunity to guide their own learning with exploration and collaboration. As the following quote from a middle school teacher shows, this in turn led them to value students' capacity to come up with their own solution methods:

> *So [the PD] taught me how to just let the kids explore, and not feel like I need to be in charge, and making sure they do it this certain way, and seeing how they can come up with the answer in an entirely different way than I might have. (Grade 7/8 teacher)*

This willingness to be open to children's approaches to solving problems, even when those ways are different from the teacher's own instinctual approach, or the textbook's prescribed approach, is a hallmark of responsive teaching. Once teachers orient themselves towards valuing student thinking in this way, they can begin working towards figuring out how to use teaching moves to help the whole class construct important mathematical understandings from those learner ideas.

- Teachers recognized the importance of building a common understanding, both of tasks and of other's mathematical ideas.

Ensuring that students were able to make sense of both the task and each other's ideas was an important component of their instructional vision for more than half of the participants. Some teachers talked about what happens before students solve a problem, citing the need to connect to a student's prior knowledge and experience as well as drawing out any questions or confusions they may have.
As the following teacher leader noted, taking time to make sense of the problem leads to more students who can get started working right away.

> *Having at least a little bit of discussion beforehand and setting up the problem with what people know and what people [have questions about], how important that actually can be, so that you will walk around and see fewer blank pages. (Math Lead)*

Other teachers focused on making sense of a person's ideas, whether that is the teacher understanding a student or students understanding each other. Following a student's thinking meant that the teacher could assess their students' understanding of the task. For example, one first grade teacher came to the

realization that a lot of students might not understand another student's explanation but that by posing questions, the teacher can help establish clarity for the whole class.

- Teachers believed in the importance of students taking on more of the cognitive load during problem solving.

As the following quote illustrates, teachers came to recognize the value of students being more actively involved in the learning process.

> *You want to give them hints. You want to give them peer interaction so that it's more self-discovery. You want to heighten their degree of self-discovery, because there's nothing like the light bulb. When you got it, you got it. You know, it's like teaching someone to fish as opposed to giving them a fish [and] it's so much more important to teach them how to fish. (Grade 6 teacher)*

A vision of how deep learning occurs and the teacher's role in that process was articulated by about a third of the participants. The idea that math was more than just a correct answer and that students should be able to explain their thinking and justify their answer was mentioned by two teachers. Others talked about the importance of allowing students the opportunity to productively struggle while solving a problem. As one 7th/8th grade teacher shared, the learning that happens when students have the opportunity to struggle to get to a solution "lasts longer". Participants also shared the value of working hard to find a solution, because students learn persistence and are proud of their work.
Some teachers began to examine their own role in the process of facilitating productive struggle. They began to specify ways that teachers could support students to deepen their learning. Several mentioned the need to hold back from their desire to help a student to allow them the opportunity to work through their challenge. When they did need to step in, it would often be to ask questions or provide a small amount of help instead of giving an answer. Allowing students to discover new knowledge themselves or in collaboration with peers also emerged as a priority in teachers' instructional vision. One teacher also mentioned the importance of pushing students to prove or generalize their findings after finding a solution:

> *The teacher's not just going to say, "Yes, that's right or wrong." So having to prove how they know the problem, they know the answer, makes them hold that accountability. I know this is right, because I did this. And I think that that's a big thing because it's really hard as a teacher not to just say, "Oh yeah, you're good. You got that." So giving them the chance to sit there and think, "Well, is there something else I could do? How could I add more to this? What can I do that makes this better?" (Grade 1 teacher)*

- Teachers valued multiple strategies for solving problems.

Almost two thirds of the participants indicated that they valued the different strategies students used to solve problems. For some, this involved recognizing there are many different ways to solve a problem. As one teacher remarked, "The one thing that I recognize about math is that we all do it different[ly]." Some teachers verbalized the importance of allowing students to keep trying, even when their strategy was different than the one the teacher would use. One teacher noticed that making connections with another person's method would help students develop more than just a procedural understanding.

Just thinking about how you got to the answer, why this person got a different method but the same answer, and I think all of those things will help them become better math thinkers, not just math repeaters. (Grade 3 teacher)

Implementation of RMT in the classroom

Although the professional development was designed primarily to give teachers a new and richer understanding of what it meant to engage in mathematical problem solving, nearly all of the classroom teacher participants described ways in which they began implementing RMT instructional practices in their own classrooms-- a change we did not necessarily expect to see until their second year of involvement. Continual reflection on their own instruction while engaged in learning was at the root of this change. One teacher noted,

Once you get that feeling of, 'Oh, this is how it's supposed to feel, this is how it's supposed to look,' then when you bring it back into your classroom you can kind of tell. Is that how it looks in here? What am I striving for? How am I going to get my classroom to kind of equal what we're doing at the training? (Grade 1 teacher)

Participants implemented different parts of the RMT instructional model and tried out various responsive teaching practices in their own classrooms, including the deliberate launch of tasks, the incorporation of more cognitively demanding tasks, and eliciting and representing multiple strategies. In addition, the 7 teacher leaders who were released from the classroom described specific ways they had implemented RMT into their own coaching practices.

- Teachers were more intentional in launching tasks.

Many of the participants mentioned being more intentional about eliciting students' questions and activating their prior knowledge when presenting students with mathematical tasks. A first grade teacher remarked, "I really try to get them to understand in the very beginning. It's making sure everyone is on the same page, making sure they understand exactly what the question is asking." Teachers noted that an inadequate launch could impact the rest of the lesson and remarked on the advantages of devoting time to establishing individual and collective understanding before students began tackling a math problem. In many cases, they described specific moves they had seen the facilitator use, such as reading a problem multiple times.

I am allowing the children to see a problem in more ways than one. So, there is the individual read. I will read. We'll have the computer read, our Smartboard, and then I'll have some collaboration with the whole group, and even adding a little peer turn and talk. That's a lot of ways to receive one question, and then to get feedback for clarity adds more to that. (Grade 6 teacher)

- Teachers incorporated rich math tasks and opportunities for productive struggle into math instruction.

More than half of the teachers mentioned incorporating more rigorous open-ended tasks requiring productive struggle into their math instruction. Teachers often took problems from existing classroom resources and adjusted them to improve their rigor or open-endedness. Although teachers acknowledged that responding to struggling students without lowering the cognitive demand of a math task was hard

work, they valued the deeper understanding that occurred as a result of productive struggle. A fourth grade teacher remarked, "I like the productive struggle. I like the concept…[The students] know that they've been working hard, they've been putting a lot into this so when they do get it, they *get* it." Participants also made an effort to take the responsive facilitation they experienced in RMT Math Circles and apply it as they worked with struggling learners. Knowing when to intervene and how much support to provide requires challenging in-the-moment decision making; one teacher observed,

> *What I like, what [the facilitator] did, was all the information came from the room. All the solutions to the problems came from the room. I have to kind of learn from that, not to give them the information, or I'll just ask probing questions to lead them so that they can lead themselves to the answer or to the solution of their problem. (Grade 7/8 teacher)*

- Teachers had students share and make sense of each other's problem solving strategies.

Eleven of the participants also mentioned using facilitation strategies from RMT Math Circles to help their students make sense of each other's work. One middle school teacher began encouraging her students to solve problems in different ways and explain their thinking, which she acknowledged was a big change in her approach: "Letting kids do different ways of answering a problem, I've never really [done] that as much. Like I would say, tell us how you got it but never like actually get up to the board and show how." In addition to displaying student work for the class to analyze and having them explain their solution processes, teachers also mentioned having one student explain another student's solution strategy and drawing connections among different problem solving methods. As one teacher explained:

> *When we finish and we come back together and they're putting it up on the board so we can work on it and they can show what they did, they have to explain, but it's not always them explaining. A lot of times, I pick somebody else. 'Why did you think that he did this or she did this? How do you know?' And if they're not quite sure, then the other kids will give them help. (Grade 1 teacher)*

Certain parts of the RMT instructional model, such as taking time to intentionally launch a task and allowing students to struggle with challenging problems, were more likely to be tried in this first year. Using a whole class discussion to share student work and make student thinking visible to all learners was mentioned less often, and the idea of connecting students' ideas to the mathematical goal of the lesson was only mentioned once. This uneven take-up suggests that some parts of the instructional model may be more easily implemented by teachers who are new to this style of teaching.
While most participants came away from the first year of RMT PD implementing one or two practices from the RMT instructional model, three participants mentioned trying to recreate a fuller experience with their students, including a launch, a task requiring productive struggle, and opportunities to make student thinking visible and draw connections to a mathematical goal.

- Teacher Leaders began to implement ideas from RMT into their coaching and classroom practice.

The teacher leaders who were released from regular classroom instruction had slightly different take-aways from the professional development. Six of them described trying out things they had learned from RMT in their own professional development or coaching work. A math coach described how the structure

of the Math Circles had influenced an ongoing professional development she was running for the teachers at her school.

> *When I'm introducing all the different instructional practices to the teachers, I lead them in that way. I make them acquire their own knowledge…I just give them a problem and just solve it, do it which way you can. And then, have them develop their knowledge and come at the end, and like, what was our big idea? What are we learning from this? And guide them through it that way. (School-Based Math Coach)*

Another math lead described implementing RMT practices when he was working with small groups of students during state testing review-- a high-stakes time when many teachers revert to prescriptive teaching methods.

> *I had an experience whereas [state testing] time came closer, I was working with a handful of small groups of students. We'd do some of the open-ended [state testing] practice, but I ran it just as [the RMT facilitator] would, basically. And that was fascinating, to actually be able to go through that whole process, especially with a small group of kids... It was kind of fun to do, to take something like the [state testing] open-ended questions and make it actually fun and exciting...All of that is putting the cognitive load back on the students. We have to know what we're doing as teachers, but we want to have… a lot of that load put on the students so that we can see what they're understanding, what they're knowing. (Math Lead Teacher)*

As this quote illustrates, participants could name and apply broader principles they learned from the experience of being a problem solver (e.g., "putting the cognitive load back on the students") to different aspects of their teaching and leadership practices.

The Emerging Gap between Instructional Vision and Practice

Despite being able to successfully implement many aspects of RMT in practice, a tension emerged for many teachers between their new or developing vision for math instruction and their ability to translate that vision to the classroom. For some teachers, this resulted in a recognition that they needed more knowledge and skills in order to be successful. For others, the perceived challenges presented barriers to believing that the vision could be realized.

- Teachers recognized that they needed more knowledge and new skills to teach mathematics responsively.

Two teachers expressed doubts about their own capacity to enact RMT stemming from their own weakness in mathematical content knowledge. One teacher commented on the difference between the math instruction she experienced as a child and responsive math instruction, noting:

> *How we learned is very procedural. This is what you do, this is what you do, this is what you do. Here's the end answer. Where now...this is what you do, and then why. I've never been taught the why. (Grade 3 teacher)*

While another teacher appreciated the value of the RMT instructional model, she struggled to implement the instructional practices with the young students in her kindergarten classroom. She described positive

interactions with the RMT facilitator as she solved math tasks herself, commenting, "I love how [the RMT facilitator] never makes you feel like you're wrong. It's like you have something. You just got to pull it out, put the pieces together." However, she admitted that while she now had that goal in mind, she did not always know how to create similar positive interactions when her own students were working on a math task:

> *I struggle because there will be kids that will say something bonkers… I don't want to say "That's not right" because [the RMT facilitator] never says, "You're not right." She says "What if?" or "How?"...Just the way she talks to you, I really would love to work on. ...How do I take those skills that I'm getting and work with six-year-olds? (Grade 1 teacher)*

Experiencing a change in vision before having the teaching skills to implement that change seemed to prime participants for a deeper focus on the implementation of more specific teaching practices.
Three of the teacher leaders mentioned the importance of making sure all of the teachers in their schools had the opportunity to learn RMT and the importance of consistency in language and messaging. As one math lead noted, "So for those who had the PD, they have been transferring what they've learned into the classroom but for those who haven't, that's the discrepancy."
Much like some teachers who realized that they needed more support to implement RMT in their classrooms, leaders also noticed the challenge in helping others to learn responsive teaching and three of them were looking forward to more support in that area in the upcoming year. A math coach who was giving a professional development to her teachers said:

> *I tried to implement a lot of the techniques that the facilitator had been teaching us. I think I still need some work. My being a scribe while people are talking and making sure that you get exactly what they are saying, I mean, that's a skill*.

Similarly, a math lead who did a lot of individual classroom coaching at her school wasn't sure she was ready to coach a teacher in RMT.

> *I'm all about learning stuff and then turning around and implementing it, that was my biggest thing. I love PD that's practical. This is super practical and hands on, but I haven't fully seen how I would help a teacher fully implement it yet*.

- Teachers had a new appreciation for the importance of planning out lessons carefully before teaching.

About a quarter of the teachers noted the necessary planning that was needed in order to teach responsively. Some teachers took note of the facilitation during the Math Circles, and realized that a great deal of planning had taken place ahead of time in the form of anticipating learners' strategies, thinking about the types of questions you might ask to help a learner get unstuck, and having a plan to reach the math goal of the lesson.

> *But [the facilitator] was so well prepared that if any one group was at a certain point, she had already anticipated that. That anticipation process that she had cultivated prior to the actual meeting, I mean, that left a huge impact on me. We can't just go in. I tell the teachers all the time. It's great if you feel like, "Oh, I'm just going to go in and teach today," but no one really can teach well like that. It takes such planning and preparation. (School-Based Math Coach)*

Other teachers reflected on their own planning and had realizations about how planning could help them support diverse learners in reaching their goals. One teacher referred to planning a responsive math lesson as creating a "blueprint of how you are going to lead everyone through the problem." Several teachers felt that they needed more time and support in order to learn how to effectively plan for this type of teaching. Teachers mentioned challenges with anticipating the types of misconceptions students might encounter, coming up with questions to base the discussion around, and understanding the instructional model well enough to plan for all parts of the lesson. Overall, there was a consensus that planning needed to take on a larger role in teachers' instruction than it currently did.

> *I'm surprised that this would not only be just planning but also that insightful part where the productive struggle part comes in and that's planned for. I know usually where my kids are going to mess up if they're multiplying, I know that they're going to try to add first. I know that that's what they're usually going to do. And I felt like the facilitator had planned for a dozen different errors, which was something I hadn't experienced. (Grade 2 teacher)*

- Some teachers perceived challenges to implementation as barriers.

One teacher described how perceived external demands to get through the curriculum made it hard for her to implement the kinds of problem solving she wanted to see in her classroom:

> *I've got to get all these topics in before the end of the year. I had to get these topics done before they had testing. I don't have time for them to problem solve…. That's where we, as teachers, become fearful and we back off. (Grade 6 teacher)*

Three teachers questioned the ability of some or all of their students to learn in the open, problem-solving environment of responsive mathematics teaching. Many studies have established a link between a teachers' beliefs and expectations for their students and student outcomes (Ferguson, 1998; Rist, 2000; Rosenthal & Jacobson, 1968). One teacher tried launching a problem but struggled to get students to see the value in taking time to make sense of the problem before solving it. This led to some classroom management issues, and the teacher wondered about using RMT with a "less challenging class." Two other teachers suggested that RMT would be better for more advanced students. A math lead stated, "It's a great method, however my concern is how effective will it be with the percentage of students in your classroom [lacking] the foundational skills?" Another teacher seemed to agree with this assessment when she said that she was focused on providing problem solving opportunities, specifically for students on the "higher academic side." A concerning finding was that a subset of teachers believed that engaging in collaborative problem solving and thinking and reasoning deeply about mathematical concepts was only accessible for some, rather than all, of their students.

Discussion

Engaging as learners in solving challenging math tasks was a powerful experience for all the participants. The teachers developed a new understanding of what it meant to learn mathematics and engage in productive struggle from a learner's point of view and this in turn gave them a new perspective on the teacher's role. A full year of Math Circle professional developments gave teachers a wide range of experiences and a new frame of reference from which to draw on to think about what it means to teach math responsively. This is particularly important when teachers have not had that kind of experience in their own K-12 education. As one participant remarked, "I've never been taught the why." As David Cohen (1990) argued over three decades ago, reforming mathematics instruction is "a very tall order," as

teachers “have to learn a new practice of mathematics teaching, while learning the new mathematics and unlearning the old." (p. 327). The unlearning and relearning that occurred for teachers over the year was therefore an important step towards creating a new illustration or *representation* of the learning process (Grossman et al., 2009) that included peer interaction and collaboration and productive struggle. Teachers developed confidence in their own ability to do math, but also understood what it felt like as a learner to be stuck or lack confidence in one’s own ideas or methods.

Many teachers were also able to translate specific practices they saw the facilitator doing into their own instruction. Teachers talked about opening up their practice to allow students to use multiple solution methods and implementing the facilitator’s methods for launching tasks to establish individual and collective understanding. This may have been aided by the fact that in each session, participants were asked to identify, name, and analyze the facilitator’s teaching moves and practices, thereby *decomposing* the RMT instructional model into its constituent parts (Grossman et al., 2009). It is also important to note that there was no pressure or expectation for teachers to change their instructional practice, so this was occurring as a result of changing visions of mathematics teaching and learning. However, because we did not observe their classroom practice, we cannot make any claims about the resulting quality of the instructional changes they reported making.

These professional learning experiences by themselves were not enough to create dramatic changes in teacher’s classroom practice. Although nearly all of our participants (93%) who taught in their own classroom described trying out something they learned, most did not implement the full instructional model, and some described challenges. Teachers did, however, express a sense of disequilibrium that was created when they perceived a gap between their evolving vision for math instruction and what they felt they were capable of enacting in their own classroom. Prior research has shown that a teacher’s instructional vision is often a precursor to their practice, as ideas about what good teaching looks like develop before the ability to bring them to life in the classroom (Munter & Correnti, 2017). This suggests the need for continual professional development to continue to support this next phase of learning, as teachers work to bring this vision into reality with their students in the classroom.

Taken together, our findings illustrate an important lesson learned from implementing this model of professional development whose resources are taxed so substantially that sustained opportunities for professional growth in mathematics teaching are not a reality. As teachers engaged in deep and meaningful inquiry around mathematics, they also engaged in inquiry into their own beliefs and practices around teaching mathematics, and this led to a desire for more learning opportunities and support for classroom implementation. In other words, learning a little bit deeply led to a desire to change practice and a felt need for more learning resources. This highlights the need for sustained opportunities for teachers to learn not just about high-quality math instruction, but to learn within their own classroom practice.

Take-Aways

- Engaging in learning mathematics through a responsive teaching model is a generative learning experience for teachers and provides a foundation for instructional vision and implementation of new practices.
- Constructing a new vision for learning and teaching mathematics does not necessarily mean that teachers know how to put that vision into practice.
- Once buy-in is achieved through establishing a new instructional vision, an important next step is learning to plan, implement, and reflect on responsive teaching in the classroom.

Questions for Further Study

This study raises some questions for further study.

- In what ways will teachers continue to draw on this experience as they move to the second year of professional development, focused on classroom implementation?
- What role does mentoring and "at the elbow" support for classroom implementation play in changing teacher's practices to align with their vision?

References

Cobb, P., & Smith, T. (2008). The challenge of scale: Designing schools and districts as learning organizations for instructional improvement in mathematics. In T. Wood, B. Jaworski, K. Krainer, P. Sullivan, & D. Tirosh (Eds.), *International handbook of mathematics teacher education.* Rotterdam, Netherlands: Sense Publishers.

Cohen, D. K. (1990). A revolution in one classroom: The case of Mrs. Oublier. *Educational Evaluation and Policy Analysis, 12*(3), 311-329.

Ferguson, Ron. (1998). Teacher perceptions and expectations and the Black–White test score gap. In C. Jencks & M. Phillips (Eds.), *The Black–White test score gap*, pp. 273–317. Washington, DC: Brookings Institution Press.

Grossman, P., Compton, C., Igra, D., Ronfeldt, M., Shahan, E., & Williamson, P. (2009). Teaching practice; A cross-professional perspective. *Teachers College Record*, *111*(9), 2065-2100.

Kazemi, E., Lampert, M., & Franke, M. (2009). Developing pedagogies in teacher education to support novice teachers' ability to enact ambitious instruction. In R. Hunter, B. Bicknell & T. Burgess (Eds.), *Crossing divides: Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia.* (Vol. 1, pp. 12-30). Palmerston North, NZ: MERGA.

Kennedy, M. M. (1999). The role of preservice teacher education. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the learning profession: Handbook of teaching and policy* (pp. 54-86). San Francisco, CA: Jossey Bass.

Munter, C., & Correnti, R. (2017). Examining relations between mathematics teachers' instructional vision, knowledge, and change in practice. *American Journal of Education*, *123*(2), 171–202.

Munter, C., Stein, M. K., & Smith, M. S. (2015). Dialogic and direct instruction: Two distinct models of mathematics instruction and the debate(s) surrounding them. *Teachers College Record*, *117*(11), 1–32.

Rist, R.C. (2000). HER classic: Student social class and teacher expectations: The self-fulfilling prophecy in ghetto education. *Harvard Educational Review, 70*(3), 257-301.

Rosenthal, R. & Jacobson, L. (1968). Pygmalion in the classroom. The Urban Review, September: 16-20.

Appendix A

Table 1. Demographics of Participant Schools in 2018-19 and 2019-20

School	No. of Participants	Grade Levels	Students Enrolled	Performance Tier* (1 low-4 high)	**Climate (1 low-4 high)	PSSA Math % proficient & advanced
A	1	K-8	450-500	2	2	9%
B	1	K-8	400-450	1	1	11%
C	2	K-8	250-300	2	2	10%
D	2	K-8	>500	2	2	12%
E	3	K-8	200-250	2	2	12%
F	1	K-4	200-250	3	4	50%
G	5	K-8	>500	4	4	77%
H	2	K-8	250-300	2	2	23%
I	1	K-8	400-450	1	1	4%
J	5	K-8	400-450	2	1	8%
K	3	K-8	>500	3	3	12%
L	2	5-8	250-300	3	4	20%
M	2	K-8	250-300	3	4	25%

*School performance representing combined performance in the Achievement, Progress, and Climate domains.

**The climate domain reflects student attendance and mobility, student and parent/guardian engagement, and student and parent/guardian perceptions of the school's climate.